High frequency data
High frequency modeling and analysis

MASEF
Slides Part II
Order book dynamics

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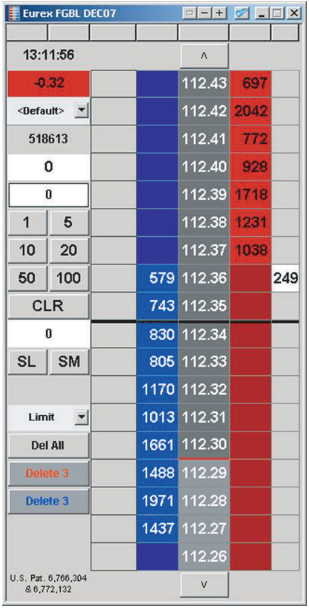
An orderbook snapshot



Basic notions

- Bid/ask spread
- Different types of orders
  - Limit orders
  - Cancel orders
  - Marker orders (generally better to send a limit order !)
- Different rules for execution matching
  - FIFO
  - Proportional
  - ...
  - Mixed rules

$\Longrightarrow$ Replayer : hard task

It's show time!

Movie showing two order books (both "large" ticks)

- Bund
- Future on EuroStoxx

More movies !

- "Kerviel" day
  - open/close $= -6.2\%$
  - open/high $= 0.3\%$
  - open/low $= -7.6\%$
- and the day after ...
  - close/open $= -2\%$
  - open/close $= 3.4\%$
  - open/high $= 4.5\%$
  - open/low $= -3.9\%$

Table 5.4. Some data for the two liquid French stocks in February 2001. The transaction volume is in number of shares.

Quantity	France-Telecom	Total
Initial/final price (Euros)	90-65	157-157
Tick size (Euros)	0.05	0.1
Total # orders	270,000	94,350
# trades	176,000	60,000
Transaction volume	$75{,}6 \times 10^6$	$23{,}4 \times 10^6$
Average bid-ask (ticks)	2.0	1.4

Bouchaud J.-Ph. and Potters M.
Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

Some Statistical elements : Some basic proxies of Liquidity

For the BUND (1 month) :

- Average Spread : $\simeq 1.02$
- Numbers of transactions : $\simeq 350000$
- Numbers of limit orders (2×4 levels) : $\simeq 1.800.000$
- Numbers of cancel orders (2×4 levels) : $\simeq 1.000.000$

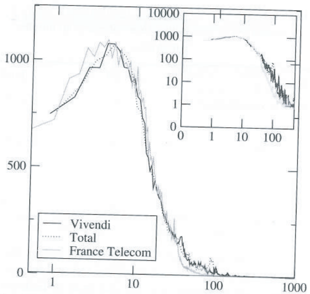


Fig. 5.3. Average size of the queue in the order book as a function of the distance from the best price, in a log-linear plot for three liquid French stocks. The 'buy' and 'sell' sides of the distribution are found to be identical (up to statistical fluctuations). Both axis have been rescaled such as to superimpose the data. Interestingly, the shape is found to be very similar for all three stocks studied. Inset: same in log-log coordinates, showing a power-law behaviour.

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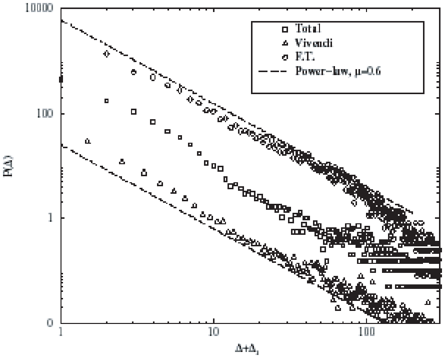
Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

Some Statistical elements : Order book profile



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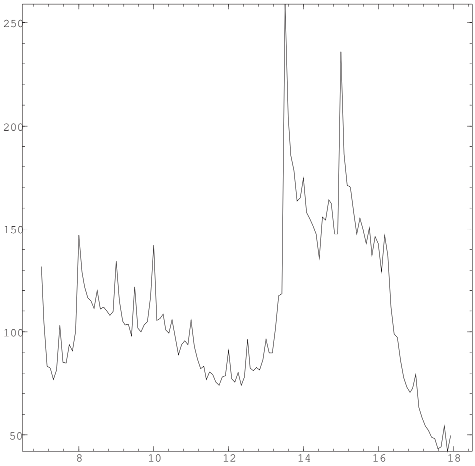
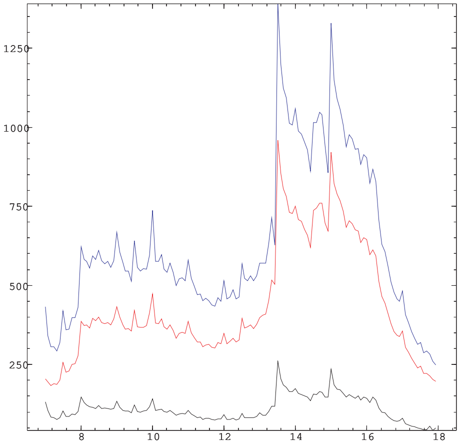
Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

Some Statistical elements : Heavy intraday seasonality



BUND 2007 : # Market orders ($\Delta t = 5$mn)
Data from BNP-Paribas FIRST-ETG, London.

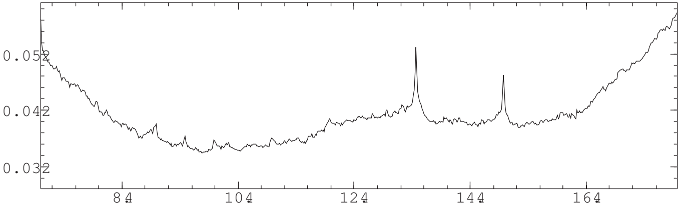


BUND 2007 : # orders ($\Delta t = 5$mn)

blue : limit orders, red : cancel orders, black : market orders.

Data from BNP-Paribas FIRST-ETG, London.

Some Statistical elements : A "better" proxy of Liquidity



BUND 2007 "Liquidity"

Tick size is 0.01

Worst price for buying/selling 600 shares within $\Delta t = 5$mn

Data from BNP-Paribas FIRST-ETG, London.

Transaction costs

- broker's fee (depends on "your size")
  - individuals $\simeq 1\%$ on stocks !
  - e.g., (small hedge fund) : 1/4 tick for futures (SXE and FX)
- bid/ask spread : cost of a buy/sell strategy
- Market impact

Spread as transaction cost : A naive arbitrage strategy

At high frequency : "large tick" $\longrightarrow$ very strong mean reversion !
$\Longrightarrow$ build an arbitrage strategy ?

- The model :

Ask price at time n : a_n
Bid price at time n : $b_n = a_n - 1$
The model for the dynamics :

$$a_n = a_{n-1} + Ne_n$$

where the $e_n \in \{-1, 1\}$ are known to be strongly anti-correlated.
At each time n, we play the mean-reversion strategy.

A naive arbitrage strategy

N	P_-	$\#samples$	P^{th}
1	80%	17600	100%
2	60%	1830	75%
3	55%	680	66.6%
4	52%	374	62.5%
5	53%	228	60%
6	53%	145	58.3%

Bund February 2007, Data BNP-Paris Bas FIRST-ETG, London.

Some order book models

- 0-intelligence (Farmer etal. 2005)
- Cont, de Larrard (2012)
  - "Level 1" model
  - Spread is 1 tick
  - Successive iid processes
- Huang, Lehalle, Rosenbaum (2013)
  - "Level n" generalization of Cont de Larrad

$\Longrightarrow$ Some related Key questions

- Execution of a limit order
- Link macro quantities (e.g., volatility) with micro parameters
- Market impact
- Market making

Impact

Key question :

How does an order impact the price ?

- Capacity of an arbitrage strategy
- Optimal execution

A naive proxy for the impact : the Response function

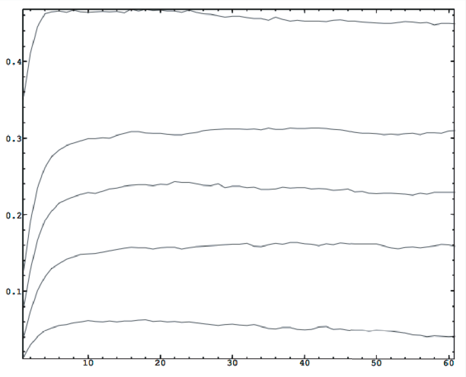
- n : trading time (i.e., n^{th} transaction) or physical time
- ϵ_n : Sign of the transaction
  $+1$ if buy market order and -1 if sell market order
- v_n : The volume of the transaction

A naive proxy for the impact :

- Response function

$$R_{n,v} = E_k\left(\epsilon_k(P_{k+n} - P_k)|v_k = v\right)$$

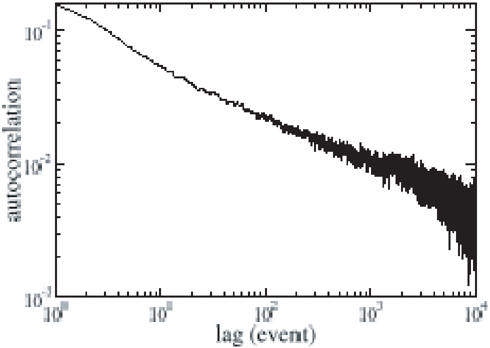
A naive proxy for the impact : the Response function



Bund Future February 2007 : Response function $R_{n,v}$
Data BNP-Paribas FIRST-ETG, London.
Volume buckets are power of 2
$\Longrightarrow$ Strong permanent impact ? ?

Some Statistical elements : Correlation of the signs of the trades

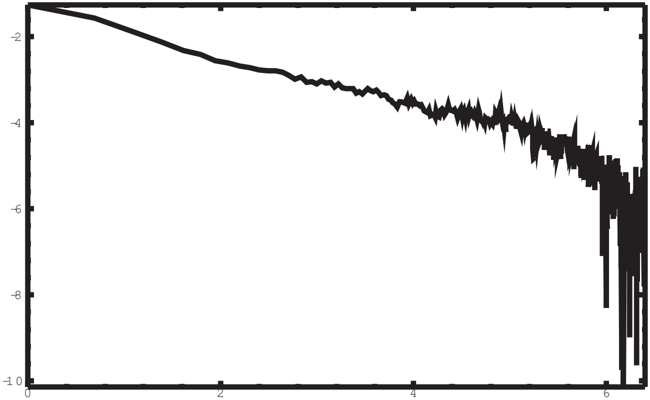
Auto-correlation function $Cor(\epsilon_k, \epsilon_{k+n})$



Bouchaud J.-Ph. and Potters M.
Theory of Financial Risk and Derivative Pricing : From Statistical

Some Statistical elements : Correlation of the signs of the trades

Auto-correlation function $Cor(\epsilon_k, \epsilon_{k+n})$ for the Bund



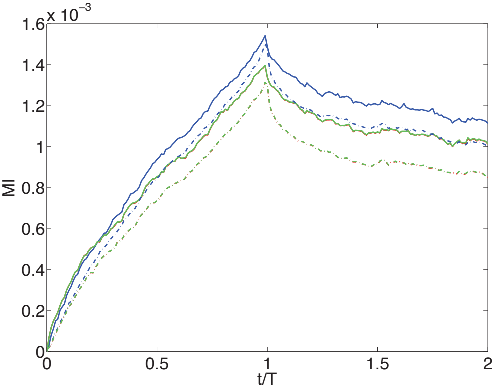
Data from BNP-Paribas FIRST-ETG, London.

Market impact profile curve

- Market impact profile of a (meta)order : variation of the price during and after the execution of the (meta)order

- Market impact profile is generally estimated by aggregating all executions of a certain type (unconditionnally to the market conditions) after time and price rescaling

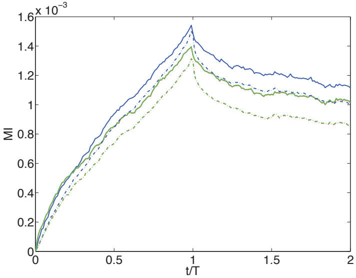
Market impact profile curve



E.B., A.Iuga, M.Lasnier, C.A.Lehalle (Estimation on a pool of European stocks
Data from CA Chevreux.

See also : Moro et al. (2009)

Market impact - "Stylized facts"



- Concave impact while trading
- Top point (temporary market impact) - "Square-root" law (Gatheral, 2008)

$$I \simeq C\sigma\sqrt{\frac{v}{V}}$$

- Relaxation after trading
- Is impact permanent ?

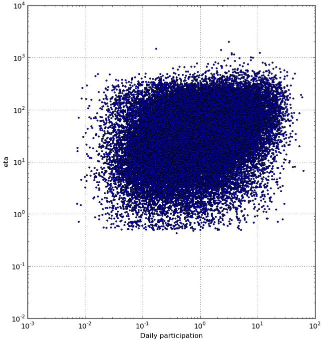
Market impact profile estimation : WARNINGS

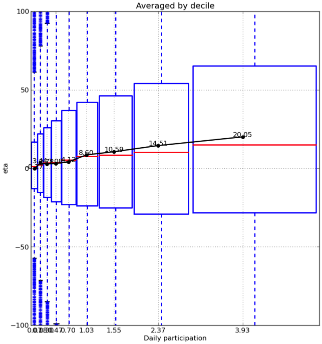
- WARNING : confusion between market impact and response function !
  $\Longrightarrow$ a priori one needs labelled data

- WARNING : Very hard to estimate due to correlation of orders arrivals and price movements

E.B., A.Iuga, M.Lasnier, C.A.Lehalle

84794 meta-orders on all European stocks from Chevreux
VWAP or Percentage of Volume strategies





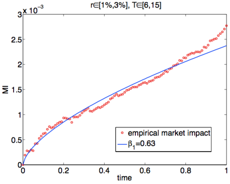
Market impact estimation : Temporary market impact

E.B., A.Iuga, M.Lasnier, C.A.Lehalle

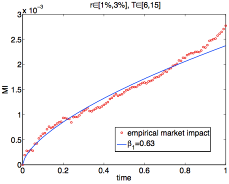
- v : Volume of the meta-order
- T : Duration of the meta-order
- V : Daily volume

$$I \simeq \left(\frac{v}{TV}\right)^{0.7}$$

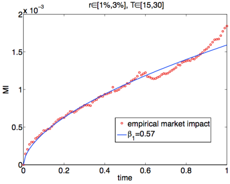
E.B., A.Iuga, M.Lasnier, C.A.Lehalle



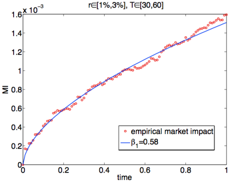
(a) fit exponent 0.96



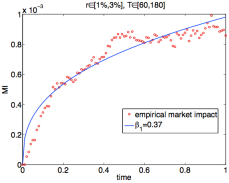
(b) fit exponent 0.63



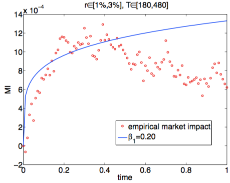
(c) fit exponent 0.57



(d) fit exponent 0.58



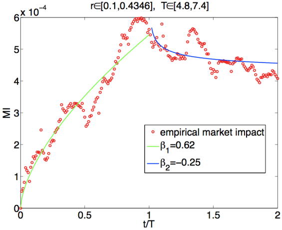
(e) fit exponent 0.37



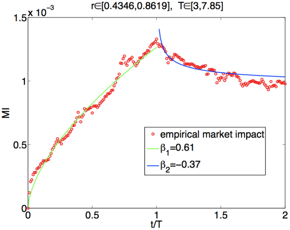
(f) fit exponent 0.21

Market impact estimation : Decay profile

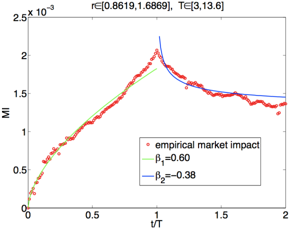
E.B., A.Iuga, M.Lasnier, C.A.Lehalle



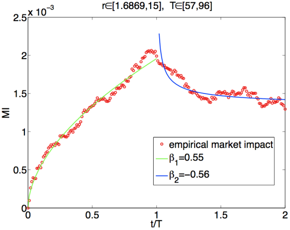
(a) $\beta_1 - \beta_2 = 0.87$



(b) $\beta_1 - \beta_2 = 0.98$



(c) $\beta_1 - \beta_2 = 0.98$



(d) $\beta_1 - \beta_2 = 1.01$

Market impact estimation : permanent impact

E.B., A.Iuga, M.Lasnier, C.A.Lehalle Permanent impact of Trend followers

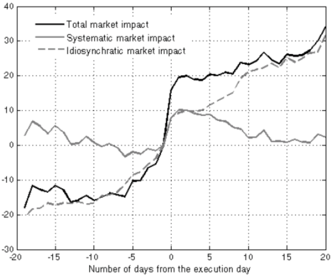


Figure 3.8: Market impact extraday for trend-follower investors (in spreads).

Market impact estimation : permanent impact

E.B., A.Iuga, M.Lasnier, C.A.Lehalle Permanent impact of Mean reverters

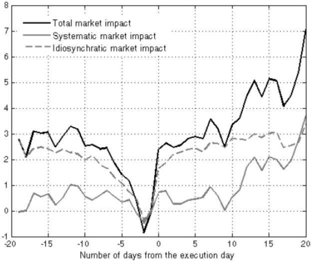


Figure 3.9: Market impact extraday for investors playing mean reversion (in spreads).